Accurate dB-Linear Variable Gain Amplifier With Gain Error Compensation

Inyoung Choi, *Student Member, IEEE*, Heesong Seo, *Student Member, IEEE*, and Bumman Kim, *Fellow, IEEE*

***Abstract*—This paper describes use of a novel exponential approximation for designing dB-linear variable gain amplifiers (VGAs). The exponential function is accurately generated using a simple error-compensation technique. The dB-linear gain is controlled linearly by the gate voltage, resulting in a simple and robust VGA. The proposed dB-linear VGA fabricated in a 65-nm CMOS process achieves a total variable gain range of 76 dB and dB-linear range greater than 50 dB with ±0.5-dB gain error. Under a 1.2-V supply voltage, the current consumption of the VGA is 1.8 mA and that of the output buffer is 1.4 mA. The input-referred in-band noise density is 3.5 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ and the in-band OIP3 is 11.5 dBm. Due to the very simple circuit topology, the total active area of the VGA and the output buffer is extremely small, 0.01 mm^2.**

***Index Terms*—Baseband amplifier, dB-linear, exponential approximation, gain error compensation, linear-in-dB, variable gain amplifier (VGA).**

I. INTRODUCTION

VARIABLE gain amplifiers (VGAs) are an important building block of wireless communication systems. The main function of a VGA is to provide a fixed output power from a large different input signal level, increasing the dynamic range of the entire system. The dB-linear gain characteristic is required for the VGA to maintain an uniform loop transient response and settling time in an automatic gain control (AGC) loop [1] and to prevent a resolution problem of control voltages for a wide variable gain range. For most applications of VGAs, the dB-linear characteristic should be accurate across a large signal range with a small gain error [2], [3]. Although many techniques have been employed to generate the exponential function, these techniques require complex circuitry with extra chip areas [4]–[7].

This paper describes a new design method involving approximation for generating the exponential function without use of any additional circuits, resulting in a simple and robust VGA. The proposed approximation can be realized using any linear term raised to the power of n. Therefore, we can implement the dB-linear characteristic by cascading n VGA stages with a linear slope. The implemented dB-linear VGA has three cascaded VGA stages and an output buffer. The approximation function of the VGA is very accurate across a wide dB-linear range owing to the proposed gain error-compensation technique. The dB-linear gain is linearly controlled by the gate bias of the control loop circuits, which is very simple.

The remainder of this paper is organized as follows. Section II lists conventional dB-linear VGAs and classifies them based on their exponential approximation methods. The proposed exponential approximation is introduced in Section III, and a gain error-compensation technique for a wide dB-linear range and a small gain error is presented in Section IV. Circuit implementation of the dB-linearity using gain error-compensation techniques is presented in Section V. The measurement results of the dB-linear VGA are summarized in Section VI. Finally, we present our conclusions in Section VII.

II. CONVENTIONAL DB-LINEAR VGAS

One of the critical issues in dB-linear VGA design is building a dB-linear gain characteristic. With a bipolar junction transistor (BJT), a dB-linear VGA can be easily designed using its exponential characteristic [8]–[10]. However, using MOS devices, it is difficult to obtain a dB-linear function with the inherent square-law and linear characteristics. Although a dB-linear VGA using a MOS device in subthreshold region has been reported [11], it can be used to limited applications owing to its large noise contribution.

Various techniques that use the inherent square-law or the linear characteristic of MOS devices have been reported for approximating the exponential function. Pseudo-exponential approximation can be expressed as

$$e^{2x} \approx \frac{1+x}{1-x} \tag{1}$$

where the relative error of (1) is less than 5% for $-0.32 \leq x \leq 0.32$ [4]. The single VGA using the ratio of the transconductance following the square root of (1) has a limited small gain range, and this topology requires use of many cascaded VGA stages to achieve a wide variable gain range. Owing to this gain limitation, it has a limited bandwidth, large chip area, and high power consumption. Another dB-linear VGA topology based on (1) is realized by tuning the feedback resistance of the VGA using MOS devices in triode region or a resistor network. However, this dB-linear VGA using MOS devices in the triode region has drawbacks of a limited control range and gain non-

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The authors are with the Department of Electrical Engineering and the Division of Information Technology Convergence Engineering, Pohang University of Science and Technology (POSTECH), Gyeongbuk 790-784, Korea (e-mail: b2dragon@postech.ac.kr; bmkim@postech.ac.kr).

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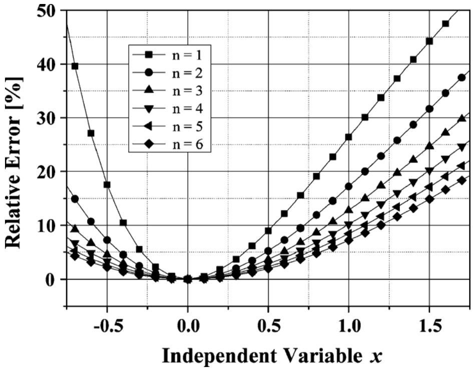


Fig. 1. Relative error of the proposed exponential approximation.

TABLE I
APPROXIMATION ERROR OF THE PROPOSED EXPONENTIAL FUNCTION

n	**equation**	x **less than** 5% **error**	x **range**
1	$1+x$	-0.28 to 0.35	0.67
2	$(1+\frac{x}{2})^2$	-0.41 to 0.48	0.89
3	$(1+\frac{x}{3})^3$	-0.52 to 0.58	1.1
4	$(1+\frac{x}{4})^4$	-0.6 to 0.67	1.27
5	$(1+\frac{x}{5})^5$	-0.68 to 0.75	1.43
6	$(1+\frac{x}{6})^6$	-0.75 to 0.81	1.56

linearity. In [5], an attempt was made to overcome these drawbacks by substituting the MOS resistors with the resistor network. For accurate and smooth gain control, the resistor network-based topology requires the use of multiple resistors and switches in each gain stage and an extra differential ramp control voltage generator. The differential ramp control voltage generator should be designed to supply the accurate control voltages to each switches of the VGAs. For a 10-ramp design, the dB-linear VGA with two gain stages requires 40 metal lines to connect between the control circuits and VGAs, and it increases in proportion to the number of the ramp signals and gain stages.

An exponential approximation based on Taylor's series is also studied extensively. The second-order expansion can be expressed as

$$e^x \approx 1 + x + \frac{1}{2}x^2. \tag{2}$$

The approximation error of (2) is less than 5% for $-0.575 \leq x \leq 0.815$ [4]. Circuit implementation based on (2) using a CMOS process requires use of various extra blocks such as current square circuit (CSC) and linear V–I converter. The extra circuits increase the chip size, power consumption, and design complexity [6], [7]. To convert a linear control voltage to an exponential gain control signal, the linear V–I converter generates a linear current output following the control input voltage and the CSC generates a squared current output using the linear current input from the V–I converter [12], [13]. The sum of resulting squared current and dc current can approximate an exponential function required for the VGA control voltage.

III. PROPOSED EXPONENTIAL APPROXIMATION

Here, we propose a new exponential approximation for a dB-linear VGA. The expression for this approximation consists of linear equation terms only and can be easily implemented as a circuit. The exponential function can be described in the limit definition as

$$e^x = \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n \tag{3}$$

where x is an independent variable and n is an integer. As n increases and x decreases, the approximation error decreases rapidly and becomes negligible. The proposed exponential approximation is expressed as

$$e^x \approx \left(1 + \frac{x}{n}\right)^n, \qquad \text{for} \quad x \approx 0. \tag{4}$$

In the proposed dB-linear VGA, the independent variable x and the output of the approximation can be defined as the gain control input x and the resultant gain, respectively. Fig. 1 shows the relationship of the relative approximation error for various x and n. The ranges of x for various values n with a relative error less than 5% are listed in Table I.

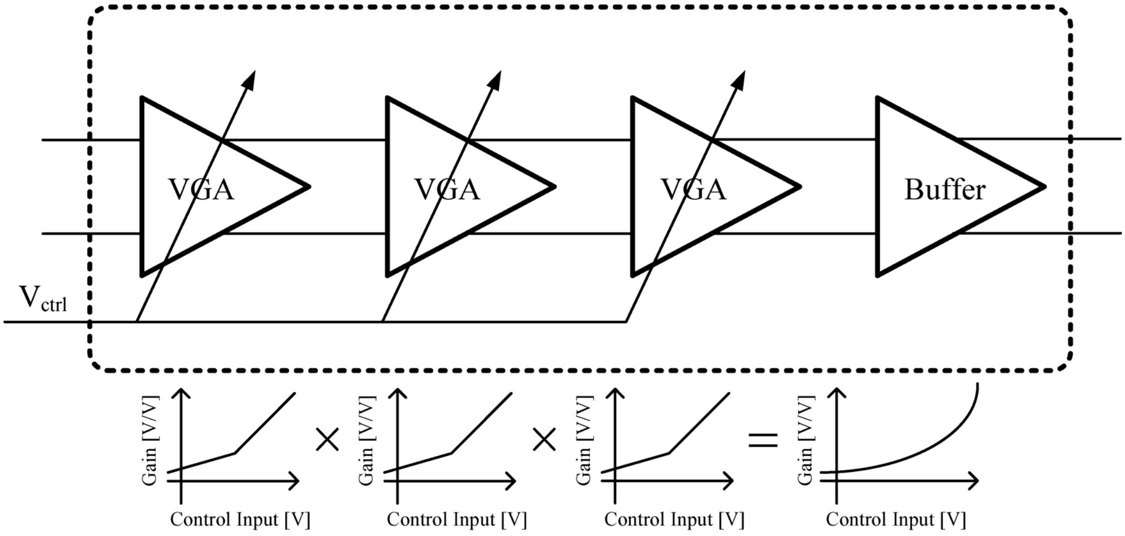
Equation (4) consists of a linear term $(1 + x/n)$, raised to the power of n. The general linear equation can be expressed as

$$y = a(x + b) \tag{5}$$

where a is the slope of the linear equation and b is the offset in the negative x direction. By raising (5) to the power of n, we have

$$\begin{aligned} a^n(x+b)^n &= a^n(x+b+n-n)^n \\ &= a^n n^n \left(1 + \frac{x+b-n}{n}\right)^n \\ &\approx a^n n^n e^{x+b-n}, \qquad \text{for } x \approx n-b. \end{aligned} \tag{6}$$

As we can see in (6), the proposed exponential approximation can be obtained by assigning an index to the general linear equations. To decrease the exponential approximation error, the n should be large. However, in a practical implementation, the value of n cannot be large, and we select $n = 3$, as shown in Fig. 2. To further reduce the error, an additional gain error compensation technique is employed. In Section V, we will discuss this compensation technique.



$$a(x+b)\times a(x+b)\times a(x+b) = a^3(x+b)^3 \approx 27a^3e^{x+b-3}$$

Fig. 2. Block diagram of proposed dB-linear VGA.

IV. GAIN ERROR COMPENSATION

In the dB-linear VGA, the available gain control input and resultant gain ranges are limited by the approximation error and the gain degradation of the single VGA stage with a large control input. To obtain a wide and accurate dB-linear characteristic, a gain error-compensation technique is developed. To this end, more than two linear terms are parallel connected in each gain stage. Each linear term approximates the same exponential function but over the different input and output regions, extending the available input and output dB-linear ranges.

The linear terms of the gain stage can be generated by adjusting the values of a and b. In (6), the approximation error is small when the value of x is close to $n - b$, and we can adjust the available control input range by changing b as

$$\begin{aligned} b' &= b - \Delta b \\ a^n(x+b')^n &\approx a^n n^n e^{x+b-n-\Delta b}, \qquad \text{for } x \approx n - b + \Delta b. \end{aligned} \tag{7}$$

As b decreases, the control input range for the available dB-linear gain shifts toward the $+x$ direction by an amount equal to Δb, but the available dB-linear gain range and the size of available control input range are not changed as shown in Fig. 3(a). The gain slope a of the linear gain stage changes the available dB-linear gain range without causing any change in the available control input range, position, and relative approximation error of the output [see Fig. 3(b)]. The varying a can be described as

$$\begin{aligned} \left(\frac{a'}{a}\right)^n &= e^{\Delta a} \\ a'^n(x+b)^n &\approx a^n n^n e^{\Delta a} e^{x+b-n}, \qquad \text{for } x \approx n - b. \end{aligned} \tag{8}$$

Equation (8) shows the available dB-linear gain range becomes $e^{\Delta a}$ times larger than (6). Using (7) and (8), the input and output ranges of the exponential approximation region can be changed.

The goal of the proposed gain error compensation is not only to increase the available dB-linear gain range but also to maintain a proper available control input range without any control voltage resolution problem. For this purpose, the proposed gain error compensation technique employing (7) and (8) should approximate the same exponential expression over the different regions. To get the desired two linear terms, we set a reference exponential equation using a general approximation term given in (6). A modified approximation term for different point can be generated by adjusting the values of a and b. The value of Δa in (8) can be determined from the desired size and position of the available dB-linear gain range. However, the modified approximation should be moved in x direction by Δa to fit the reference exponential equation [see Fig. 3(c)]. The movement of $\Delta a = \Delta b$ can be expressed as

$$\begin{aligned} a'^n(x+b')^n &\approx a^n n^n e^{\Delta a} e^{x+b-n-\Delta b} \\ &\approx a^n n^n e^{x+b-n}, \qquad \text{for } x \approx n - b + \Delta b. \end{aligned} \tag{9}$$

The modified approximation equation in (9) is the same as (6), but has a different available control input and dB-linear gain ranges, shifted by Δa in the x-direction and $e^{\Delta a}$ in gain, respectively. Therefore, we can expand simultaneously the available control input and linear-dB gain ranges by employing the two linear terms

$$A_1 = a_1(x+b_1) \tag{10}$$

$$A_2 = a_2(x+b_2) \tag{11}$$

$$A_{\text{total}} = (A_1 + A_2)^n \tag{12}$$

where A_1 and A_2 are the two parallel linear terms for the small control input and large control input, respectively. The total gain A_{total} in (12) is obtained by raising the summation of the two different linear terms in (10) and (11) to the power of n. For a small control input, the linear term for the large input A_2 can be negligible in the total gain A_{total} because A_2 is almost zero for the control input $x \leq b_2$. Therefore, the standard design can be

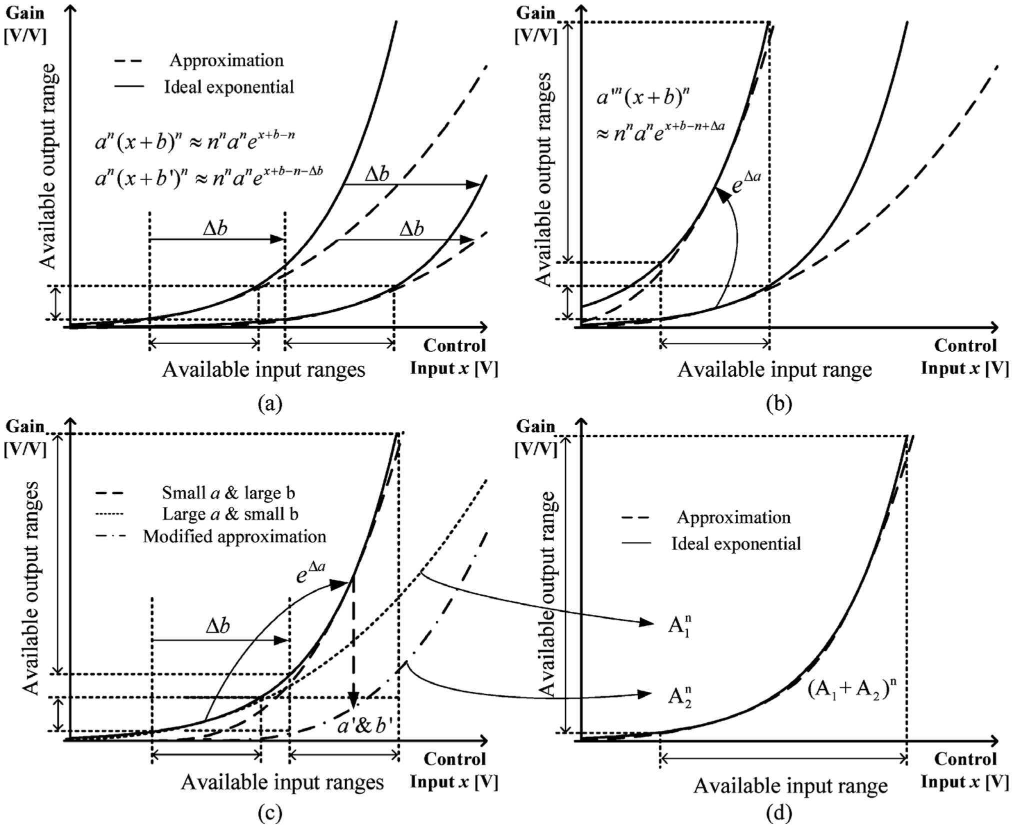


Fig. 3. Steps in proposed gain error compensation process. Response of approximation function versus (a) b value and (b) a value. (c) Approximation functions for small and large control inputs x. (d) Combined response of the two approximation functions.

employed for A_1 representing the low gain region. However, for a large control input, the terms for the small and large control inputs have significant values and the linear term for the large control input should be modified by adjusting the values of a_2 and b_2 in (11). As shown in the Appendix, the a_2 and b_2 should be changed as follows:

$$a_2' = a_2 - a_1 \tag{13}$$

$$b_2' = \frac{a_2 b_2 - a_1 b_1}{a_2 - a_1}. \tag{14}$$

The gain obtained from the two linear terms at the intermediate region between the two available ranges is slightly smaller than the original exponential function. Thus, we can achieve the available control input and dB-linear gain ranges that are wider than the summation of all available ranges obtained from the two approximation exponential terms, as shown in Fig. 3(d). We can employ more than two linear terms using the same method for better approximation with a smaller error. To increase the available dB-linear gain range, we should use larger a. However, the gain slope of the linear VGA stage has a limit, and a very steep gain slope causes resolution problem for the control voltage.

V. Circuit Implementation

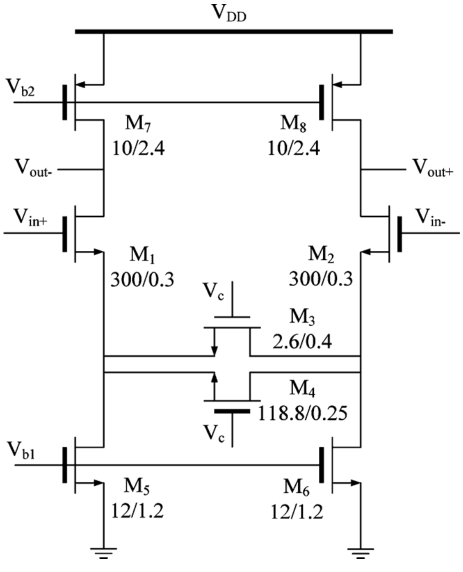


Fig. 4. Schematic of a VGA stage with two linear terms. The transistor with a thin gate line is a thin oxide device and the numbers are W/L ratios.

Using the proposed approximation for the exponential function in conjunction with the gain error-compensation technique, we realize a new dB-linear VGA. We employ three cascaded VGA stages with the two linear terms. Fig. 2 shows a block diagram of the proposed VGA comprising the three cascaded VGA stages with a buffer. Although increasing the number of cascaded VGA stages and the linear terms results in reduced approximation gain error and maintains the dB-linear characteristic over a wider control input range, for the demonstration purposed, we have employed three VGA stages, each of which

TABLE II
PERFORMANCE SUMMARY AND COMPARISON

	[14]	[11]	[15]	[16]	[17]	[6]	[10]	[18]	[5]	This work
Gain Range [dB]	0 to 70	-35 to 55	-42 to 42	-10 to 36	-10 to 20	0 to 60	-39 to 55	-13.5 to 13.5	-18 to 47	-13 to 63
dB-linear Range [dB]	-	80	-	-	-	-	79.4	27	40	50
Gain Error [dB]	-	3	-	-	-	-	3	1	0.8	0.5
Bandwidth [MHz]	20	30 - 210	20 - 350	15	20	2.87	4 - 900	380 - 2200	40	14.8
OIP3 [dBm]	33	27	20	26	29.2	12	-	19	22	11.5*
IRN [nV/$\sqrt{\text{Hz}}$]	9	10	9	4.2	11.2	1.64	9	8	11	3.5*
Supply [V]	3.3	2.5	1.8	2.5	1.8	2.5	1.8	1.8	1.2	1.2
Power [mW]	33	27.2	5.4	16	2.43	11.25	20.5	19.8	2.23	3.84
Size [mm^2]	0.24**	0.49**	0.18**	0.8**	0.3**	0.6**	0.42**	0.1**	0.15**	0.01**
Process [nm]	500	250	250	130	180	350	180	180	65	65

* Reported value is for the maximum gain setting of 63 dB.

** dB-linear VGA and buffer only.

is a differential VGAs [6] with two control loops, as shown in Fig. 4. However, the number of the cascaded VGA stages and linear terms can be determined by the desired dB-linear gain range and gains of the single VGA stages. As the number of the stages or the linear terms increases the chip are and power consumption can be increased. Because of the very simple circuit structure of this VGA, the penalty is insignificant compared with the other reported ones (see Table II).

The gain of the single VGA stage can be expressed as

$$A_v = -G_{s,\text{total}} R_o \frac{g_m}{g_m + 2G_{s,\text{total}}} \tag{15}$$

where g_m, $G_{s,\text{total}}$, and R_o represent the transconductance of the input transistors M_1 and M_2, total conductance of M_3 and M_4, and output resistance of M_7 and M_8, respectively. Equation (15) can be rewritten as

$$A_v \approx -G_{s,\text{total}} R_o \tag{16}$$

where $g_m \gg G_{s,\text{total}}$. The VGA stage has two loops controlled by transistors M_3 and M_4, which have different threshold voltages and W/L ratios for the gain error compensation. Each conductance can be expressed as

$$G_{s,\text{M3}} = \mu C_{\text{OX}} \left(\frac{W}{L}\right)_{\text{M3}} (V_{\text{gs,M3}} - V_{\text{th,M3}}) \tag{17}$$

$$G_{s,\text{M4}} = \mu C_{\text{OX}} \left(\frac{W}{L}\right)_{\text{M4}} (V_{\text{gs,M4}} - V_{\text{th,M4}}) \tag{18}$$

$$G_{s,\text{total}} = G_{s,\text{M3}} + G_{s,\text{M4}}. \tag{19}$$

The gain of the single VGA stage shown in Fig. 4 is controlled by the loops

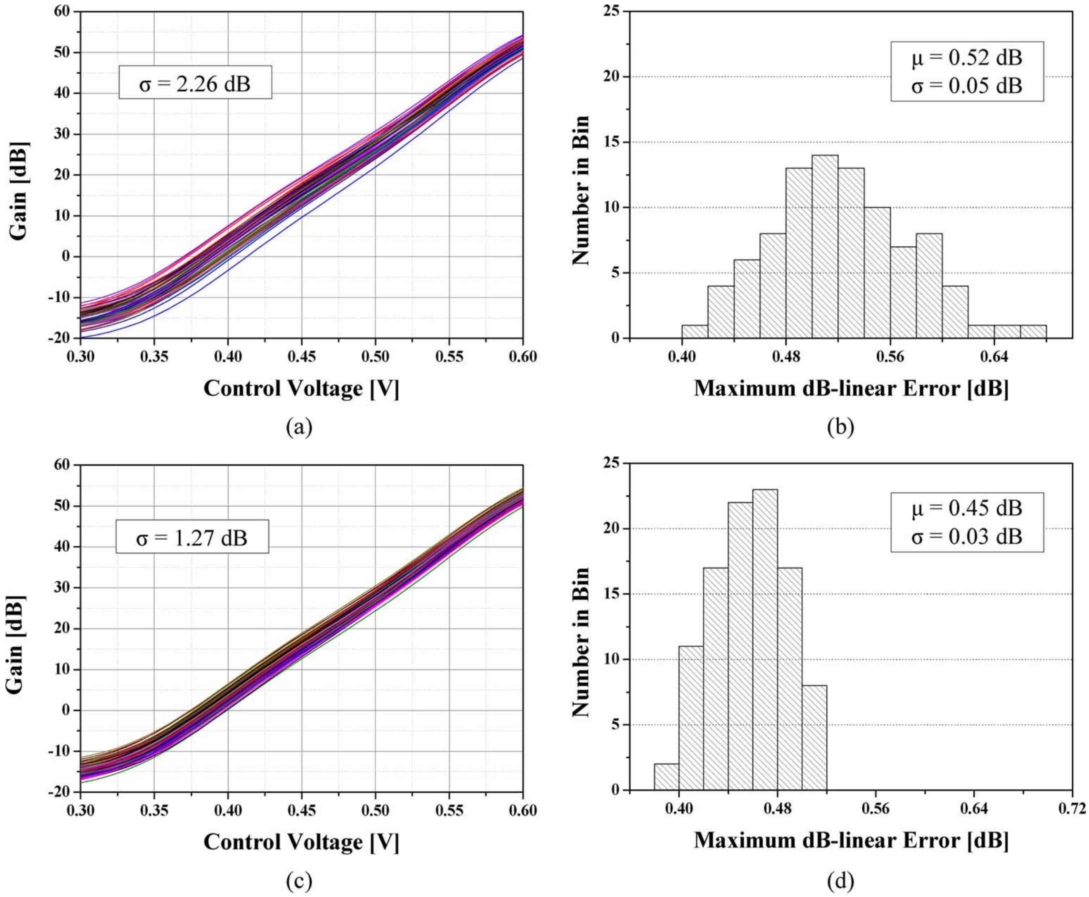
$$\begin{aligned} A_{v,\text{M3}} &\approx -G_{s,\text{M3}} R_o \\ &\approx -\mu C_{\text{OX}} \left(\frac{W}{L}\right)_{\text{M3}} (V_{\text{gs,M3}} - V_{\text{th,M3}}) R_o \quad (20) \\ A_{v,\text{M4}} &\approx -G_{s,\text{M4}} R_o \\ &\approx -\mu C_{\text{OX}} \left(\frac{W}{L}\right)_{\text{M4}} (V_{\text{gs,M4}} - V_{\text{th,M4}}) R_o \quad (21) \\ A_v &= A_{v,\text{M3}} + A_{v,\text{M4}} \quad (22) \end{aligned}$$

where $V_{\text{gs,M3,M4}} = V_c - V_{\text{DS,M5,M6}}$, $V_{\text{gs,M3}} \approx V_{\text{gs,M4}}$, and V_c is the control voltage. $V_{\text{DS,M5}}$ and $V_{\text{DS,M6}}$ are the drain–source voltages of M_5 and M_6, which remain nearly constant and equal for the differential inputs. The linear function given in (5) can be defined as

$$\begin{aligned} x &= V_c \quad (23) \\ a &= -\mu C_{\text{OX}} \left(\frac{W}{L}\right) \quad (24) \\ b &= -V_{\text{DS,M5,6}} - V_{\text{th,M3,4}}. \quad (25) \end{aligned}$$

In (24) and (25), we can adjust the values of a and b using W/L ratios and threshold voltages, respectively. For the different threshold voltages, we can select MOSFETs with different oxide thicknesses or use the body effect. In this work, we used the two transistors with different oxide thicknesses. However, the body effect method can afford greater tunability with more accurate gain error compensation. The total gain of the dB-linear VGA can now be expressed as

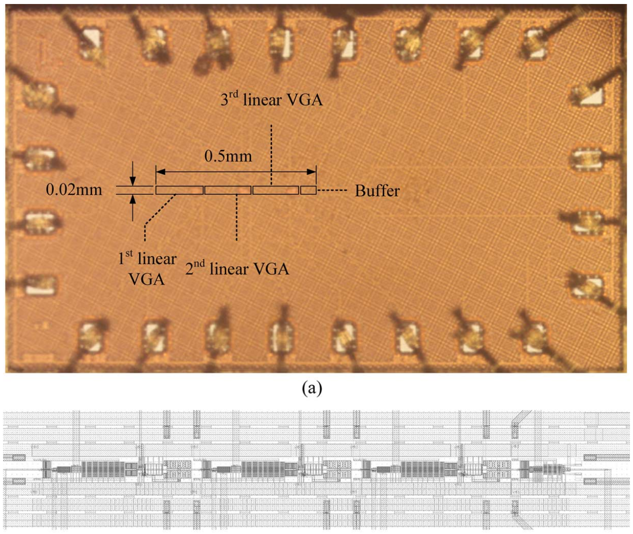
$$A_{v,\text{total}} = (A_{v,\text{M3}} + A_{v,\text{M4}})^3. \tag{26}$$



(a) (b)

(c) (d)

Fig. 5. (a) Cube of the single linear VGA gain. (b) Maximum dB-linear gain error histogram of the single linear VGA gain. (c) Gain of the proposed VGA. (d) Maximum dB-linear gain error histogram of the proposed VGA (100 Monte Carlo simulation results).



(a)

(b)

Fig. 6. (a) Die photograph and (b) layout with an active area of 0.01 mm^2.

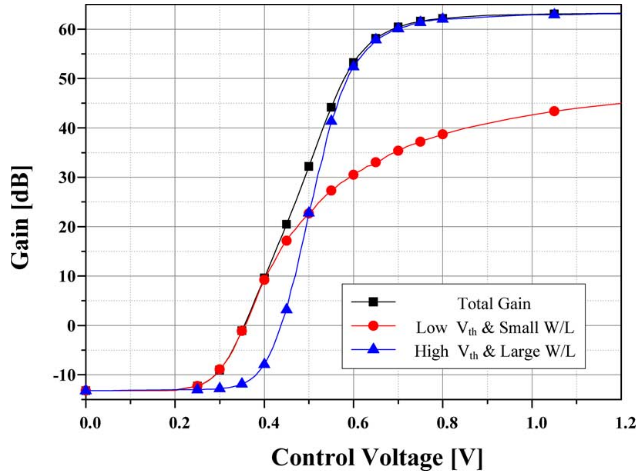


Fig. 7. Measured gain of dB-linear VGA at 4 MHz.

For a large control input, the condition $g_m \gg G_{s,\mathrm{total}}$ and (15) may not be satisfied anymore, and the gain of the VGA stages is compressed as $G_{s,\mathrm{total}}$ increases. To compensate this gain degradation, we can use larger a value than that calculated in Section IV and the Appendix.

In dB-linear VGAs, the process variation and mismatch effects are critical issues for generating the dB-linear function. The dB-linear function produced by extra control circuits is very sensitive to the process variation and mismatch because the control signal errors from the control circuits have a direct effect on the gain of the dB-linear VGA. However, the proposed VGA generates the dB-linear function by cascaded variable gain stages without any extra circuits, which is a big advantage for

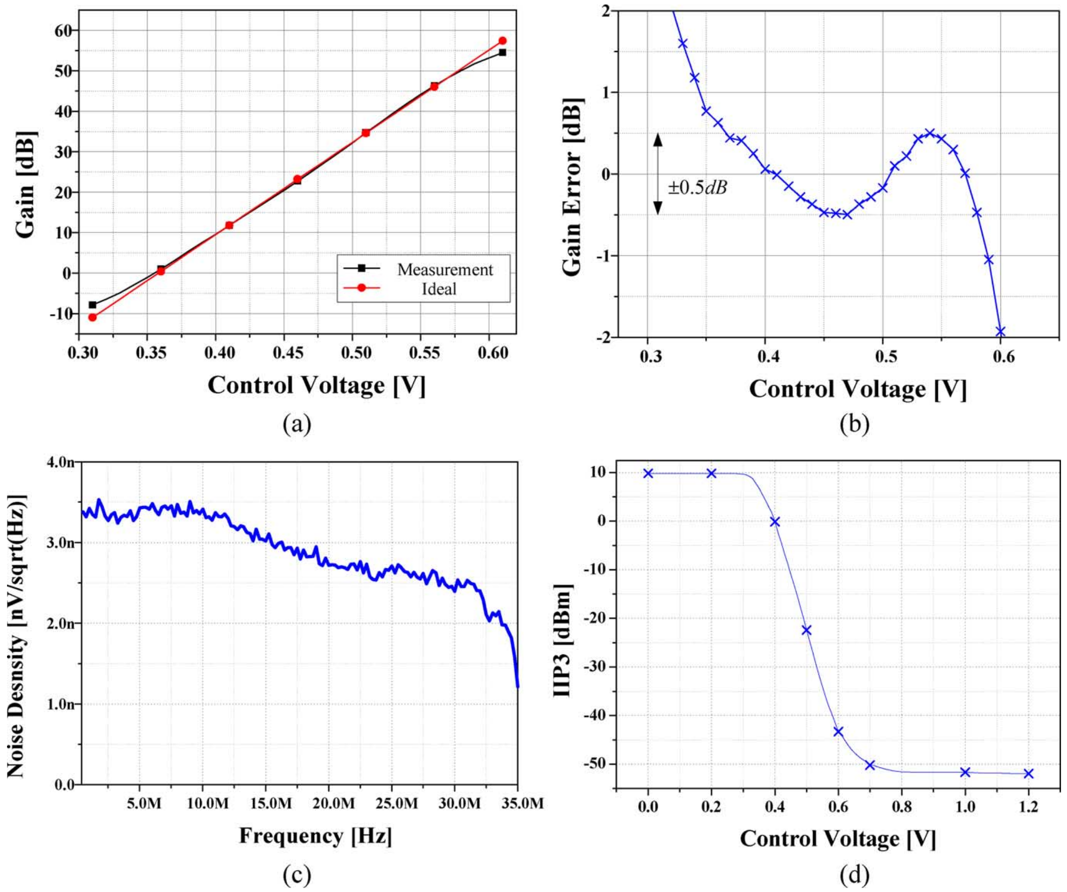


Fig. 8. Measurement results: (a) dB-linear range at 4 MHz; (b) gain error at 4 MHz; (c) input-referred noise; (d) IIP3 of two-tone test with 10- and 11-MHz signals.

simplicity and robustness. The threshold voltages and W/L ratios of the linear VGAs can be affected by the process variation and mismatch. The biased threshold voltage fluctuation due to the process variation can be easily adjusted by the control voltage but the unbiased threshold voltage and W/L ratio variation can be an issue for the dB-linear gain error. We have evaluated the both effects using Monte Carlo simulation. Fig. 5 presents the gain variations and maximum dB-linear gain errors of the proposed dB-linear VGA and the single linear VGA of the proposed VGA at 4 MHz for 100 Monte Carlo simulations. To compare the single linear VGA and the proposed VGA, we used the cube of the simulated single linear VGA gain because the proposed VGA consists of three cascaded linear VGAs. As shown in the figure, gain variations and the dB-linear gain errors of the VGA are small for both cases indicating that this VGA are robust to the process variation and mismatch. The variations for the proposed VGA is smaller than the single linear VGA, which means that, using the cascaded gain stages, the uncorrelated random variation can be compensated effectively. The maximum dB-linear gain error histograms of the single linear VGA and the dB-linear VGA shown in Fig. 5(b) and (d). The worst maximum dB-linear errors are 0.67 and 0.51 dB, and they yield σ of 0.05 and 0.03 dB, respectively. These results present the cascaded VGAs topology reduces the gain variation and dB-linear gain error together. Therefore, we can claim that the dB-linear VGA proposed here is very compact structure with simple control and robust to the mismatch.

VI. MEASUREMENT RESULTS

The dB-linear VGA was fabricated using a 65-nm CMOS process, and the total active area of the dB-linear VGA and buffer is 0.01 mm^2, which is extremely small (see Table II for comparison). The current consumption of this VGA is 1.8 mA and that of the output buffer is 1.4 mA with a 1.2-V supply voltage. A photograph and layout of the chip are shown in Fig. 6(a) and (b). For each VGA stage measurement, an additional 12 differential I/O pads are connected to inputs and outputs of the linear VGAs and buffer. The two control pads are also connected to M_3 and M_4 loops. Fig. 7 shows the measured gain at 4 MHz of the VGA for three cases, A_1, A_2, and A_{total}. For the total gain of the dB-linear VGA measurement, two control pads are connected together and the total gain range is 76 dB. Fig. 8(a) shows that the dB-linear range is greater than 50 dB with ±0.5 dB gain error when the control voltage V_c is varied from 0.36 to 0.57 V. The dB-linear range with only the M_3 transistor loop is approximately 13 dB when V_c of M_3 loop is varied from 0.36 to 0.42 V and V_c of M_4 loop is connected to ground. The dB-linear range with only M_4 loop is approximately 30 dB when V_c of M_4 loop is varied from 0.48 to 0.56 V. As discussed before, the gain slope of M_4 loop for the large input range is smaller than the total dB-linear VGA gain slope because of the M_3 loop gain. The dB-linear range of the total gain is larger than the sum of the dB-linear ranges with the M_3 and M_4 loops. Fig. 8 shows the other

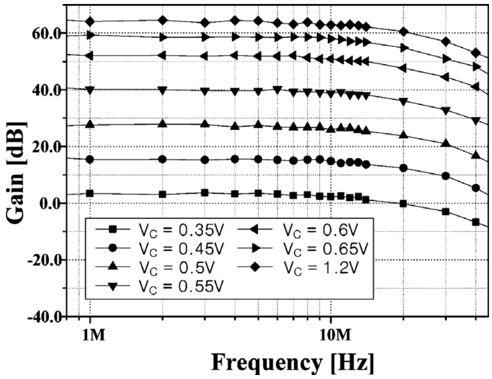


Fig. 9. Frequency responses in various control voltages.

characteristics of the VGA. The input referred in-band noise density is 3.5 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ and the in-band IIP3 versus various control voltages corresponding to the two-tone signals at 10 and 11 MHz is shown in Fig. 8(d). The in-band IIP3 is 9.8 dBm at minimum gain setting of -13.2 dB and the in-band OIP3 is 11.5 dBm at a maximum gain setting of 63.2 dB. The bandwidth of the dB-linear VGA is 14.8 MHz, but the bandwidth is limited by parasitic capacitance of the I/O pads for the various gain measurements and can be significantly larger. Fig. 9 presents the frequency responses in various control voltages. The measurement results are summarized in Table II together with previously reported results.

VII. CONCLUSION

In this paper, we introduced a novel exponential approximation method for a dB-linear VGA. The proposed approximation does not require an extra circuit for generating the exponential function and it drastically reduces design complexity and chip area. Moreover, the dB-linear gain can be controlled easily using the gate bias. Because of the simple control method, this VGA is robust to the process variation. The VGA based on the proposed exponential approximation can be fabricated using any VGA that has a linear gain characteristic. This dB-linear VGA has a gain error resulting from approximation error and the gain degradation of the linear VGA stages under a large control voltage. Therefore, the dB-linear VGA requires a gain error compensation for accurate dB-linear characteristic over wide input and output ranges. We used a simple gain error compensation technique to provide accurate exponential approximations over the small and large gain ranges of the dB-linear VGA. With the proposed gain error compensation, we achieved a dB-linear gain range greater than 50 dB within ±0.5-dB gain error.

APPENDIX
GAIN ERROR COMPENSATION AT THE LARGE CONTROL INPUT RANGE

The different linear terms for the small and large control inputs can be expressed as

$$a_1^n(x+b_1)^n, \qquad \text{for } x \approx n - b_1 \tag{27}$$

$$a_2^n(x+b_2)^n, \qquad \text{for } x \approx n - b_2' \tag{28}$$

where $b_1 > b_2$. The total gain of (12) at the large control input $x \geq b_2'$ is given by

$$[a_1(x+b_1)+a_2'(x+b_2')]^n = (a_1+a_2')^n\left(x+\frac{a_1b_1+a_2'b_2'}{a_1+a_2'}\right)^n. \tag{29}$$

Comparing (28) and (29), a_2' and b_2' for the modified A_2 should satisfy the following conditions:

$$a_2' = a_2 - a_1 \tag{30}$$

$$b_2' = \frac{a_2b_2 - a_1b_1}{a_2 - a_1}. \tag{31}$$

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Inyoung Choi (S'10) was born in Seoul, Korea, in 1984. He received the B.S. degree in electrical engineering from Pohang University of Science and Technology (POSTECH), Pohang, Korea, in 2007, where he is currently working toward the Ph.D. degree.

His research interests are CMOS RFIC and analog circuits for wireless communications, especially multi-mode and multi-application receiver design for SDR systems.

Heesong Seo (S'10) received the B.S. degree in electrical engineering from Pohang University of Science and Technology (POSTECH), Pohang, Korea, in 2007, where he is currently working toward the Ph.D. degree.

His research interests include CMOS RFIC for wireless communications, Digital RF sampling circuit for SDR system, and mixed-mode signal-processing IC design.

Bumman Kim (M'78–SM'97–F'07) received the Ph.D. degree in electrical engineering from Carnegie Mellon University, Pittsburgh, PA, in 1979.

He joined the Central Research Laboratories, Texas Instruments Inc., where he was involved in the development of GaAs power field-effect transistors (FETs) and monolithic microwave integrated circuits (MMICs). He has developed a large-signal model of a power FET, dual-gate FETs for gain control, high-power distributed amplifiers, and various millimeter-wave MMICs. In 1989, he joined the Pohang University of Science and Technology (POSTECH), Pohang, Gyeungbuk, Korea, where he is a POSTECH Fellow and a Namko Professor with the Department of Electrical Engineering, and Director of the Microwave Application Research Center. He is involved in device and circuit technology for RF integrated circuits (RFICs) and power amplifier. He has authored over 300 technical papers.

Prof. Kim is a member of the Korean Academy of Science and Technology and the National Academy of Engineering of Korea. He was an associate editor for the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, a Distinguished Lecturer of the IEEE Microwave Theory and Techniques Society (IEEE MTT-S) and an AdCom member.